

method. Comparison are sparse and  $y_1(t) = s *$ nation, MIMOSOFOM, This As Figod ap mostatterulti#path systems rropuctiond width.

es have seen their number of antennas hind multiple output systems are linked e electroladenetic (EM) multipath channess diferent channel conditions. Under al to estimate each of these channels, and hbination to achieve greater capacity than

tipath channels have a Sparse Common ne paths<sup>9</sup> Time of Arrivals (ToA) are the small error  $\leq \varepsilon$ . Under this assumption, Innovation (FRI) sampling [3] based alof the SCS property, conveniently called other algorithms which also try to estiet of Fourier "probes" (such as lowpass  $\Delta t$ ectrum), SCS-FRI has four main advantion allows for joint recovery of the supoutputs, in the pendently of the paths amof probes used to sense the channel can ndwidth for data transmition. Third, for annel estimation is more robust to noise qualization gain. Last but not least, the ed by a very small set of parameters, saveamforming. SCS channels in a discrete



Fig. 1. (a) Transmission over a medium with two scatterers and P receiving antennas. (b) The P channels contain two paths arriving at the same time up to  $\pm \varepsilon$ . The amplitudes of paths from scatterer are (possibly correlated) Rayleigh variates [9] $i, \ell [m] = \sum_{k=1}^{\infty} c_k u_k^{k}$ ,

uses baseband DFT pilots (probes) to solve the aforementioned estimation problem. Within this setup, we will derive defamér-R20 theounds (CRB) on the support estimation for both deterministic and Rayleigh fading channets.

We will then show that SCS-FRI is not restricted to baseband DFT pilots, and works equivalently well in the uniformely spaced ("scattered") DFT pilots layout ubiquitous in OFDM based communications. Like other channel destimation techniques based on cscattered pilots, the s only requirement is for the dannel impulse response (CIR) to be relatively short compared to the symbol duration.

Then we investigate the efficient use of other probing sequences. It

### Extension to MIMO case

**Proposition 2** In a MIMO system with SCS channels, let I be the number of sources, L the number of sensors and K the cardinality of the channel support. If

#### $L \ge KI$ ,

and if there exists a subband of at least K+3I frequency indices such that (15) holds for  $m_0 + 1 \leq m < m_0 + K + 3I$ , then the MIMO system can be fully resolved up to an amplitude ambiguity matrix Eand a delay ambiguity  $\tau$ .

### Low-Rate Sampling

**Proposition 3** Under the same condition as stated in Proposition 2, perfect reconstruction on all the sensor measurements can be achieved with probability one, given that we keep  $L \geq KI$  sensor samples on a subband of K + 3I frequency indices and  $L' \geq I$  sensor samples on all the other frequency indices.

Observations: the multiple channels in the output are highly correlated.

notivate the SCS channel model and prohe SCS-FRI algorithm. *I* This algorithm

partment of Electrical Engineering and tom\*  $x_i(t)$ , fornia, Berkeley, CA 94720, USA. This re-Inc., ERC Advanced Grant-Support for Fron-006 and SNF Grant - New Sampling Methods n Nr : 200021-121935/1.

is shown that they must have the same span as a subset of DFT basis vectors to warrant the use of a DFT pilot based algorithm. Interestingly, the set of Walsh-Hadamard sequences, used in most CDMA based standards, verifies this property with the added benefit of provid-

ing uniformely scattered DFT pilots. This enables the use of SCS-FRI le input systems using non-blind transmit Minipage All these equivalences allow to use the CRB derived in Section 4.

udied in [7] within the compressed sens- We conclude the study with numerical results snowing the enciency Time Domain Convolution the SCS-FRI algorithm in Fare que augustion fare put to the study with numerical results snowing the enciency setup, and the potential equalization gain compared to the industry standard which is lowpass interpolation of the CIR spectrum.

> PFS Y[m] = H[m] X[m]2. PROBLEM FORMULATION DFS

Consider a bandpass channel of bandwidth B. The inverse bandwidth 1/B sets a limit on the distance at which two pulses of bandwidth BInherent Ambiguities

The MIMO systems have inherent ambiguities:

• A *time shift ambiguity* such that if we shift input signals to some constant time  $\tau$  and shift the channel response with  $-\tau$ , the shift can not be observed in the output signals.

• A scale ambiguity such that if we rearrange the input signals with matrix E, and rearrange the channels with  $E^{-1}$ , the rearrangement cannot be observed in the output signals.



Efficient Low-rate Sampling: • Fully sampling in a small subband of the output signals and use them to recover all the channel properties • Only a sparse sensing on the frequency rest of band **1S** 

necessary to fully recover the

input signals.

### **Experimental Results**





# $\{\xi^{-m}E^{-1}X[m],\xi^{m}H[m]E\}$

# **Blind Estimation**

# Considering a SIMO case

**Proposition 1** In a SIMO system with SCS channels, if the number of sensors L is greater than or equal to the cardinality of the channel support K, i.e.,

### L > K

and if there exists a subband of at least K + 3 continuous Fourier coefficients such that  $X[m] \neq 0$  for  $m_0 \leq m < m_0 + K + 3$ , then the system can be fully resolved up to two free parameters, namely an amplitude ambiguity e and a delay ambiguity  $\tau$ .

Reconstruction result of a MIMO system. Experiments are performed 100 times at each SNR point, and the box plots show the median (red lines), 25th and 75th percentile (blue boxes) and the extreme data points (black lines).

Reconstruction result of a SIMO system by three different approaches. The first (blue, bottom) uses estimated time directly; the second (red, middle) uses optimized time by a nonlinear minimization; and the last (cyan, top) uses the ground truth time.

# Summary

• A novel algorithm solving MIMO systems with *sparse common support*.

- *Blindly estimate* the unknown source signals and the channel information by using only the sensor measurements.
- Derived an efficient *low-rate* sampling scheme that significantly reduces the number of samples.